CS42

MODEL QUESTION PAPER

USN

Fourth Semester B.E. Degree Examination, February/March 2004

CS/IS

Graph Theory and Combinatorics

Time: 3 hrs.] [Max.Marks : 100

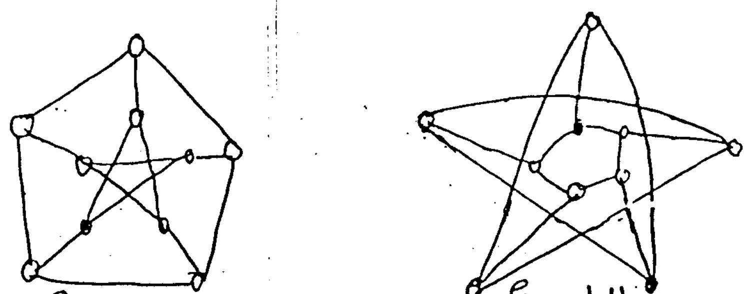
Note: ***Answer any FIVE full questions choosing atleast TWO full questions from each part.***

PART A

1. (a) If $G = (V, E)$ be a connected undirected graph what is the largest value for $|V|$ = number of vertices if $|E|$ = number of edges = 19 and deg(v)$\geq$4 for all $v \in V$? **(6 Marks)**

   (b) Prove that G has an Euler circuit if and only if G is connected and every vertex in G has an even degree for G an undirected graph without isolated vertices. **(7 Marks)**

   (c) Define isomorphism of two simple graphs. Show that in the following figure Graph G is isomorphic to graph H. **(7 Marks)**

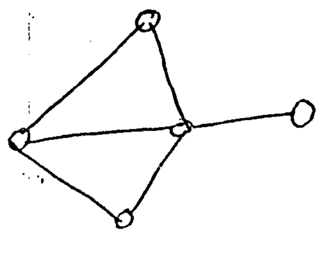


Graph G Graph H

2. (a) i) Define Bipartite graph.

   ii) Prove that the complete graph on five vertices is non planar. **(7 Marks)**

   (b) Find the geometric dual of



(6 Marks)

   (c) Using Decomposition theorem for chromatic polynomials $P(G, \lambda)$ find $P(G, \lambda)$ G a cycle of length four. **(7 Marks)**

3. (a) Prove that for a tree T on n vertices and m edges $n = m + 1$ **(7 Marks)**

Contd.... 2

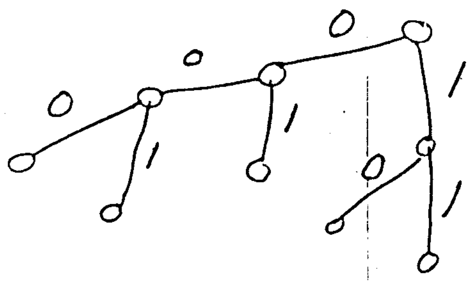
(b) i) Define prefix code

ii) Which of the following sets represent the prefix code? State reasons

$A = \{000, 001, 01, 10, 11\}$

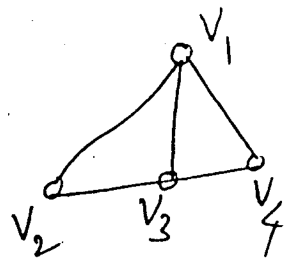
$B = \{1, 00, 01, 000, 0001\}$

iii) Obtain a binary prefix code using the labelled binary tree.



(7 Marks)

(c) Find all the spanning trees of a graph.



(6 Marks)

4. (a) Explain Kruskal's algorithm for finding minimal spanning tree of a weighted graph. **(7 Marks)**

(b) Define i) cut set,

ii) edge connectivity,

iii) vertex connectivity with one example each. **(6 Marks)**

(c) Prove that the maximum flow possible between two vertices a and b in a network (graph) is equal to the minimum of the capacities of all cut-sets with respect to a and b. **(7 Marks)**

PART B

5. (a) How many positive integers 'n' can we form using the digits 3,4,4,5,5,6,7 if we want n to exceed 5,000,000? **(6 Marks)**

(b) A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols the transmitter will also send a total of 45 (blank) spaces between the symbols, with atleast three spaces between each pair of consecutive symbols. In how many ways can tansmitter send such a message? **(8 Marks)**

Contd.... 3

(c) In how many ways can one arrange three 1's and three -1's so that all six partial sums (starting with the first summand) are nonnegative? (5 Marks)

6. (a) State and prove extended pigenhole principle. Show that if any five numbers from 1 to 8 are choosen then two of them will add up to 9. (6 Marks)

(b) Explain Rook polynomial. (7 Marks)

(c) Define derangement. In how many ways we can arrange the numbers 1,2,......, 10 so that 1 is in 1st place, 2 is not in 2nd place and so on and 10 is not in 10th place? (7 Marks)

7. (a) Determine the generating function of the numeric function

$$a_r = 2^r \text{ if } r \text{ is even}$$
$$= -2^r \text{ if } r \text{ is odd.}$$
(7 Marks)

(b) Find the generating function for the sequence

0,2,6,12,20,30,42,... (6 Marks)

(c) Using the summation operator theory find a formula to express $0^2 + 1^2 + 2^2 + \ldots.. + n^2$ as a function of n. (7 Marks)

8. (a) The number of virus affected files in a system is 1,000 and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day? (7 Marks)

(b) Solve the Fibonacci recurrence relation

$$F_{n+2} = F_{n+1} + F_n \text{ for } n \geq 0, \quad F_0 = 0, \quad F_1 = 1$$
(6 Marks)

(c) Solve the non homogenous recurrence relation

$$a_n - 3a_{n-1} = 57^n \text{ where } n \geq 1 \text{ and } a_0 = 2$$
(7 Marks)

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CS42

NEW SCHEME

USN

Fourth Semester B.E. Degree Examination, July/August 2004

Computer Science /Information Science and Engineering

Graph Theory and Combinatorics

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions, choosing atleast TWO full questions from each Part.*

PART - A

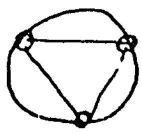
1. (a) Show that there is no graph with 12 vertices and 28 edges where,
   i) the degree of each vertex is either 3 or 4
   ii) The degree of each vertex is either 3 or 6. (6 Marks)

   (b) How many edge disjoint Hamiltonian cycles exist in the complete graph on seven vertices ? Also draw the graph to show these Hamiltonian cycles. (7 Marks)

   (c) Define isomorphism of two graphs. Give an example to show that two graphs need not be isomorphic though they have equal number of edges, equal number of vertices and equal number of vertices with a given degree sequence. (7 Marks)

2. (a) Define complete bipartite graph. Prove that Kuratowski's second graph, $K_{3,3}$ is nonplanar. (7 Marks)

   (b) Draw the geometric dual of the graph given in the following figure. (6 Marks)



   (c) Prove that the vertices of every planar graph can be properly coloured with five vertices. (7 Marks)

3. (a) Prove that a tree with n vertices has $n - 1$ edges. (6 Marks)

   (b) i) Define prefix code

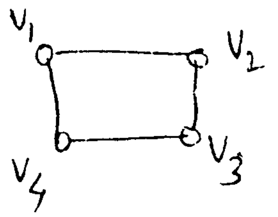
   ii) Which of the following sets represent prefix code?

   State reasons

   $A = \{000, 001, 01, 10, 11\}$

   $B = \{1, 00, 01, 000, 0001\}$ (7 Marks)

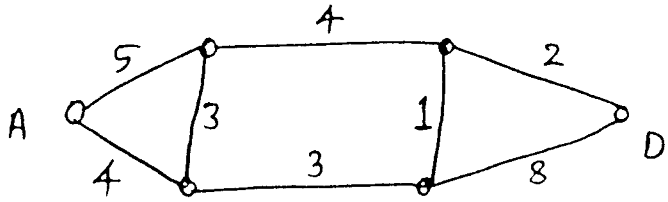
   (c) Find all the spanning trees of the graph given below. (7 Marks)



4. (a) Explain Prim's algorithm for finding shortest spanning tree of a weighted graph. (7 Marks)

   (b) Define i) Cutset, ii) Edge connectivity iii) Vertex connectivity with one example each (6 Marks)

   (c) Find the maximum flow possible between the vertices A and D for the following graph. (7 Marks)



PART - B

5. (a) In how many ways can one distribute 10 identical white marbles among six distinct containers ? (6 Marks)

   (b) A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols the transmitter will also send a total of 45 (blank) spaces between the symbols with atleast three spaces between each pair of consecutive symbols. In how many ways can transmitter send such a message ? (7 Marks)

   (c) Use Catlan numbers to find the number of ways to list eight symbols which include four 0's and four 1's so that in each case the number of 0's never exceed number of 1's. (7 Marks)

6. (a) State and prove extended Pigenhole principle. Hence show that if 30 dictionaries in a library contain a total of 61, 327 pages, then one of the dictionaries must have atleast 2045 pages. (6 Marks)

   (b) Using the principle of inclusion and exclusion determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2 or 3 or 5. (7 Marks)

   (c) Define derangements. Find the number of derangements of 1, 2, 3, 4 using exponential series technique. Also made a list of all the derangements of the above example mentioned. (7 Marks)

7. (a) Find the exponential generating function of the sequence $1, 2, 2^2, 2^3, 2^4, \ldots$ (7 Marks)

   (b) Find the number of partitions of positive integer $n = 6$ in to distinct summands as a coefficient of x^6 in the generating funtion of $p_d(6)$. Also list these partitions. (7 Marks)

   (c) What is summation operator ? Explain. (6 Marks)

8. (a) Find the generating function of the linear recurrence relation
   $$C_n = 3C_{n-1} - 2C_{n-2} \text{ with } C_1 = 5, C_2 = 3.$$ (6 Marks)

   (b) Find the generating function of
   $$a_n + a_{n-1} - 6a_{n-2} = 0 \text{ for } n \geq 2, \quad a_0 = -1 \ \& \ a_1 = 8$$ (7 Marks)

   (c) Find the generating function of
   $$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$$ (7 Marks)

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NEW SCHEME

CS42

USN

Fourth Semester B.E. Degree Examination, January/February 2005

Computer Science and Information Science Engineering

Graph Theory & Combinations

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer FIVE full questions choosing atleast TWO full question from each Part.*
2. All questions carry equal marks.

PART - A

1. (a) Define :
   i) Connected graph
   ii) Spanning subgraph and
   iii) Compliment of a graph. Give one example for each. **(6 Marks)**

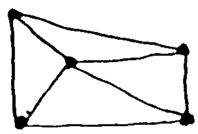
   (b) Explain with example graph isomorphism. Show that in a graph G the number of odd degree vertices is even. **(7 Marks)**

   (c) Write a note on "Konigsberg-bridge" problem. **(7 Marks)**

2. (a) Define :
   i) Planar graph
   ii) Complete Bipartite graph and
   iii) Dual of a planar graph.
   Give one example for each. **(6 Marks)**

   (b) Show that in any connected planar graph with n vertices, e-edges and f-faces $e - n + 2 = f$. (Eulers formula). **(7 Marks)**

   (c) Define chromatic number and chromatic polynomial. Find the chromatic polynomial for the graph given below. **(7 Marks)**



3. (a) Define :
   Tree ii) Binary rooted tree and iii) Prefix code.
   Give one example for each. **(6 Marks)**

   (b) Prove that a tree with n vertices has $n - 1$ edges. **(7 Marks)**

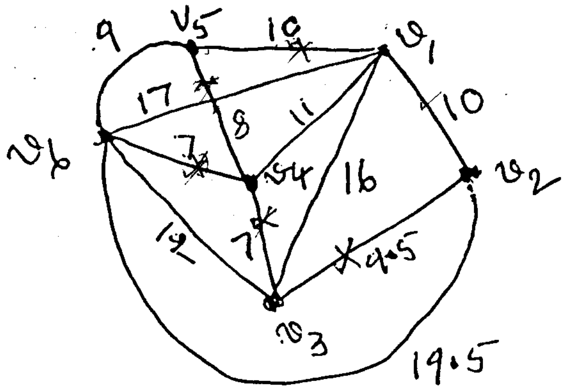
   (c) Obtain a prefix code to send the message ROAD IS GOOD using labeled Binary tree and hence encode the message. **(7 Marks)**

4. (a) Define : **(7 Marks)**
   i) Vertex connectivity ii) Edge connectivity.
   iii) Bridge iv) Cut vertex with an example.

Contd.... 2

(b) Prove that the maximum flow possible between two vertices a and b in a network (graph) is equal to the minimum of the capacities of all cut-sets with respect to a and b. **(6 Marks)**

(c) Find the shortest spanning tree using Prims algorithm for the weighted graph given below. **(7 Marks)**



PART - B

5. (a) State sum and product rule of counting. Give one example for each. **(6 Marks)**

(b) i) How many 9 letter words can be formed using the letters of the word "Difficult"? **(7 Marks)**

ii) A certain question paper contains two parts A and B each contains 4 questions. How may different ways a student can answer 5 questions by selecting atleast two questions from each part?

(c) In how many ways can one travel in the xy-plane from (0,0) to (3,3) using the moves $R : (xy) \to (x+1, y)$ and $U : (x, y) \to (x, y+1)$ if the path taken may touch but never rise above the line $y = x$? Draw two such paths in xy-plane. **(7 Marks)**

6. (a) State Pigen hole principle and generalised Pigen hole principle. Show that if any five numbers from 1 to 8 are choosen then two of them will add up to 9. **(6 Marks)**

(b) Define Rook polynomials and Forbidden positions. There are 6 pairs of students gloves in a box and each pair is of different colour. Suppose right gloves are distributed at random to six students. and then the left gloves are distributed at random to them. Find the probability that

i) No student gets a matching pair

ii) Everybody gets a matching pair. **(8 Marks)**

(c) In a Dormitory there are 12 students who take art course (A), 20 who take biology (B), 20 who take chemistry (C) and 8 who take drama course (D). There are 5 students for both A and B, 7 students for both A and C, 4 students for A and D, 16 students for B and C, 4 students for B and D and 3 students for C and D. There are 3 students who take A, B and C, 2 for A, B and D, 2 for B, C and D, 3 for A, C and D. Finally there are 2 in full four courses. It is also known that there are 71 students in the dormitory who have not signed up for any of these courses. Find the total number of students in the dormitory. **(6 Marks)**

Contd.... 3

CS42

7. (a) Define (ordinary) generating function and exponential generating function. Give one example for each. (6 Marks)

(b) Find the coefficient of x^{18} in the product

$(x + x^2 + x^3 + x^4 + x^5)\,(x^2 + x^3 + x^4 + - - - -)^5$ (7 Marks)

(c) Find :

i) The sequence corresponding to the generating function $3x^3 + e^{2x}$ (7 Marks)

ii) Generating function for the sequence 0,2,6,12,20,30,42 - - - .

8. (a) Solve the recurrence relation (Fibonacci relation) $F_{n+2} = F_{n+1} + F_n$ given $F_o = 0$ & $F_1 = 1$ and $n \geq 0$ (6 Marks)

(b) Using generating function solve :

$y_{n+2} - 4y_{n+1} + 3y_n = 0$ given that $y_0 = 2, \;\; y_1 = 4$ (7 Marks)

(c) Find the general solution of $s(k) - 3\,s(k-1) - 4\,s(k-2) = 4^k$ (7 Marks)

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NEW SCHEME

CS42

USN

Fourth Semester B.E. Degree Examination, July/August 2005

Computer Science and Information Science Engineering

Graph Theory & Combinatorics

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer FIVE full questions choosing atleast TWO full questions from each Part.***

PART - A

1. (a) Define a directed garph and an undirected graph.

   i) If $G = (V, E)$ is an undirected graph with $|V| = v$, $|E| = e$, and no loops, prove that $2e \leq v^2 - v$.

   ii) State the corresponding inequality for the case when G is directed. **(6 Marks)**

   (b) For the undirected graph in fig 1(b), find and solve a recurrence relation for the number of closed $u - v$ walks of length $n \geq 1$.

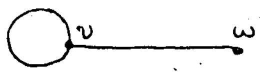

Fig. 1(b) **(4 Marks)**

   (c) Define an induced subgraph.

   i) Let $G = (V, E)$ be an undirected graph, with $G_1 = (V_1, E_1)$ a subgraph of G. Under what condition(s) is G_1 not an induced subgraph of G ?

   ii) For the graph G in Fig 1(c), find a subgraph that is not an induced subgraph. **(5 Marks)**

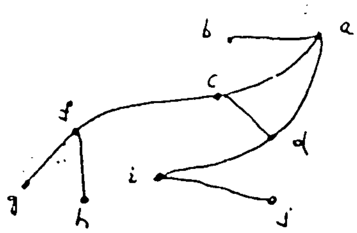

Fig 1(c)

   (d) Define graph isomorphism. Let $G = (V, E)$, $H = (V', E')$ be undirected graph with $f : V \rightarrow V'$ establishing isomorphism between the graphs.

   i) Prove that $f^{-1} : V' \rightarrow V$ is also an isomorphism for G and H.

   ii) If $a \in V$, prove that $deg(a)(inG))$. $deg(f(a))$ (inH) **(5 Marks)**

2. (a) Define a bipartite graph. Can a bipartite graph contain a cycle of odd length? Explan. **(6 Marks)**

Contd.... 2

(b) What is a Hamilton cycle ?

i) For $n \in Z^+$, $n \geq 2$, show that the number of distinct Hamilton cycles in the graph $K_{n,n}$ is $\frac{1}{2}(n - 1)!\,n!$

ii) How many different Hamilton paths are there for $K_{n,n}$ & $n \geq 1$? **(7 Marks)**

(c) Consider the graph $K_{2,3}$ shown in fig 2(c), and let $\lambda \in Z^+$ denote the number of colours available to properly color the vertices of $K_{2,3}$.

i) How many proper colorings of $K_{2,3}$ have vertices a, b coloured the same ?

ii) How many proper colorings of $K_{2,3}$ have vertices a, b colored with different colors ?

iii) What is the chromatic polynomial of $K_{2,3}$? What is $\chi(K_{2,3})$?

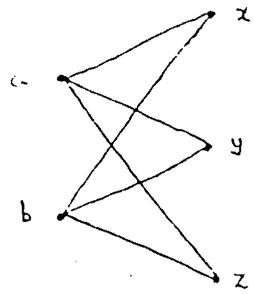

Fig. 2(c) **(7 Marks)**

3. (a) Define a tree. In every tree $T = (V, E)$, show that $|V| = |E| + 1$.

If a tree has four vertices of degree 2, one vertex of degree 3, two of degree 4, and one of degree 5, how many pendant vertices does it have?

(b) List the vertices in the tree shown in fig 3(b) when they are visited in a preorder traversal and in a post order traversal. **(6 Marks)**

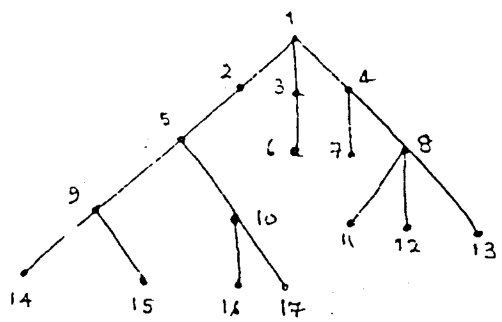

Fig. 3(b)

(c) Write Mergesort algorithm and derive its time complexity. **(6 Marks)**

4. (a) Write Kruskal's algorithm to find a minimal spanning tree for a loop-free undirected graph $G = (V, E)$ with $|V| = n$. Prove that the spanning tree obtained by this algorithm is optimal. Prove that the time-complexity of the algor thm is $O(n^2 \log_2 n)$. **(12 Marks)**

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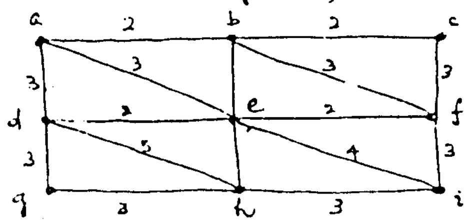
(b) Apply Kruskals algorithm to determine minimal spanning tree for the graph shown in Fig 4(b)

Fig. 4(b)



(8 Marks)

PART - B

5. (a) Let $A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6\}$
   i) How many functions are there from A to B ? How many of these are one-to-one ? How many are onto ?
   ii) How many functions are there from B to A ? How many of these are onto? How many are one-to-one ? (10 Marks)

   (b) What are catalan numbers ? Consider the moves.

   $R : (x, y) \rightarrow (x+1,\ y)$ and $U : (x, y) \rightarrow (x, y+1)$

   In how many ways can one go
   i) from (0,0) to (6,6) and not rise above the line $y = x$?
   ii) from (2,1) to (7,6) and not rise above the line $y = x - 1$?
   iii) from (3,8) to (10,15) and not rise above the line $y = x + 5$? (4 Marks)

   (c) Let triangle ABC be equilateral, with $AB = 1$. Show that if we select 10 points in the interior of this triangle, there must be at least two whose distance apart is less than 1/3. (Hint : Use Pigeonhole principle) (6 Marks)

6. (a) i) In how many ways can the letters in ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters ? at least two pairs of consecutive identical letters ? (8 Marks)
   ii) Answer part (i), replacing two with three.

   (b) Give a combinatorial argument to verify that $\forall n \in Z^+$,

   $$n! = \binom{n}{0} d_0 + \binom{n}{1} d_1 + \ldots + \binom{n}{n} d_n = \sum_{k=0}^{n} \binom{n}{k} d_k,$$

   (for each $1 \leq k \leq n$, d_k = the number of derangements of $1, 2, \ldots k; d_0 = 1$) (5 Marks)

   (c) A pair of dice, one red and the other green is rolled six times. We know that the ordered pairs (1,1), (1,5), (2,4), (3,6), (4,2), (4,4), (5,1), and (5,5) did not come up. What is the probability that every value come up on both the red die and the green one ? (7 Marks)

7. (a) For $n \in Z^+$, find in $(1 + x + x^2)(1 + x)^n$ the coefficient of (6 Marks)
   i) x^7 ii) x^8 iii) x^r for $0 \leq r \leq n+2$, $r \in Z$

Contd.... 4

(b) Determine the generating function for the sequence a_0, a_1 ...... where a_n is the number of portions of the nonnegative integer n into

i) even summands

ii) distinct even summands

iii) distinct odd summands.

(6 Marks)

(c) Define an exponential generating function (EGF). Find the EGF for the number of ways to arrange n letters, $n \geq 0$. selected from each of the following words :

i) HAWAII ii) MISSISSIPPI iii) ISOMORPHISM

(8 Marks)

8. (a) The number of bacteria in a culture is 1000 (approximately), and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day.

(6 Marks)

(b) Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$ and $a_0 = 1$, $a_1 = 4$.

(6 Marks)

(c) Solve the following recurrence relations by the method of generating functions:

i) $a_{n+2} - 3a_{n+1} + 2a_n = 0, n \geq 0, a_0 = 1, a_1 = 6$

ii) $a_{n+2} \quad 2a_{n+1} + a_n = 2^n, n \geq 0, a_0 = 1, a_1 = 2$

(8 Marks)

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NEW SCHEME

CS42

Reg. No.

Fourth Semester B.E. Degree Examination, January/February 2006

Computer Science and Information Science Engineering

Graph Theory & Combinatorics

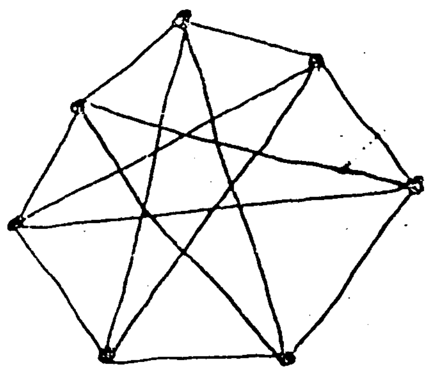
Max.Marks : 100

Note: *Answer FIVE full questions choosing atleast TWO full questions from each Part.*

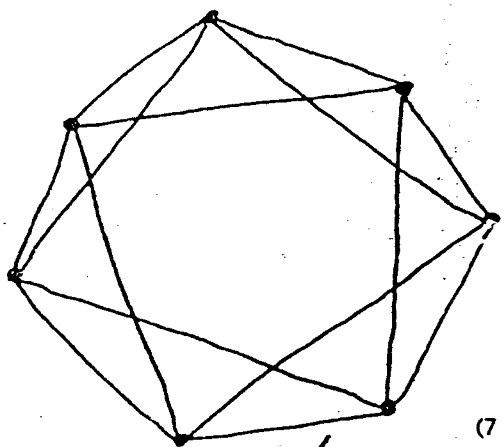
PART - A

1. (a) Determine $|V|$ for the following graphs G.
   i) G has nine edges and all vertices have degree 3.
   ii) G has 10 edges with two vertices of degree 4 and all others of degree 3. **(6 Marks)**

   (b) Define isomorphism of graphs. Show that the following two graphs are isomorphic.



G_1



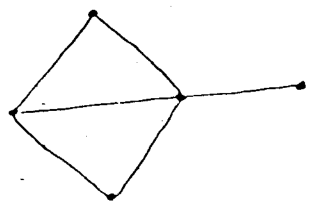
G_1'

(7 Marks)

   (c) Define Hamilton cycle.
   Prove that in the complete graph with n vertices, where n is odd and ≥ 3, there are $\frac{n-1}{2}$ edge - disjoint Hamilton cycles. **(7 Marks)**

2. (a) Define i) a planar graph ii) a bipartite graph, and iii) a complete bipartite graph. Give one example for each **(6 Marks)**

   (b) Find the geometric dual of the graph $G = (V, E)$. Write down any four observations of G and its dual.

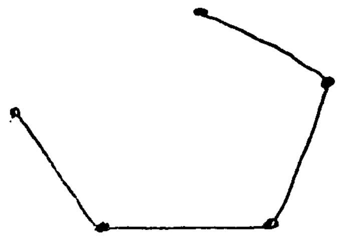


G_1

(7 Marks)

Contd.... 2

(c) Define chromatic number of a graph.
Find the chromatic polynomial $P(G, \lambda)$ for the following graph G. Hence find the chromatic number.



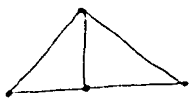
(7 Marks)

3. (a) If a tree $T = (V, E)$ has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, find the number of pendant vertices in T. (6 Marks)

(b) Construct an optimal prefix code for the symbols

a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7, respectively. (7 Marks)

(c) Define spanning tree of a graph. Find all the spanning trees of the following graph.

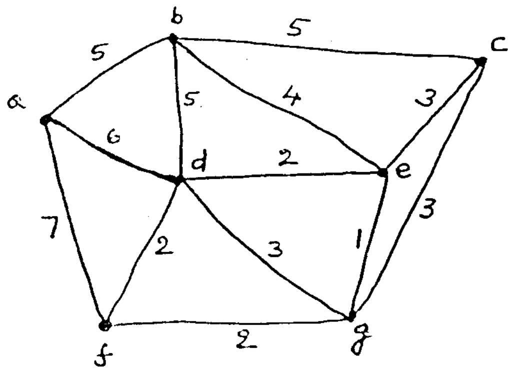


G_1

(7 Marks)

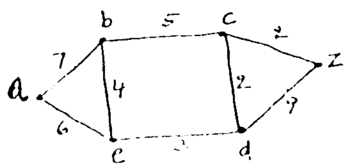
4. (a) Define i) Cutset, ii) Edge - connectivity, and iii) vertex connectivity. Give one example for each. (6 Marks)

(b) Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below :



(7 Marks)

(c) For the network shown below, find the capacities of all the cutsets between the vertices a and Z, and hence determine the maximum flow between a and Z.



(7 Marks)

Part - B

PART - B

5. (a) How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000 ? **(6 Marks)**

(b) Consider the following program segment, where i, j and k are integer variables.

```
for i: = 1 to 20 do
  for j : = 1 to i do
    for k : = 1 to i do
      print (i • j + k)
```

How many times is the print statement executed in this program segment ? **(7 Marks)**

(c) Use catalar numbers to find in how many ways can one arrange four 1's and four -1's so that all eight partial sums (starting with the first summand) are non-negative? List all the arrangements. **(7 Marks)**

6. (a) State the Pigeonhole principle and the extended pigeonhole principle. Show that if any six numbers from 1 to 9 are chosen then two of them will add up to 10. **(6 Marks)**

(b) In a certain area of the countryside, there are five villages a, b, c, d, e . An engineer is to device a system of two-way roads so that, after the system is completed, no village will be isolated. In how many ways can he do this ? **(7 Marks)**

(c) Define derangement. In how many ways we can arrange the numbers 1, 2, 3, ... 10 so that 1 is not in 1^{st} place, 2 is not in 2^{nd} place and so on and 10 is not in 10^{th} place. **(7 Marks)**

7. (a) Determine the generating function of the numeric function

$$a_r = \begin{cases} 2^r & if\ r\ is\ even \\ -2^r & if\ r\ is\ odd \end{cases}$$
(6 Marks)

(b) Use generating function to determine how many four element subsets of $S = \{1, 2, 3, ...15\}$ contain no consecutive integers. **(7 Marks)**

(c) A company hires 11 new employees, each of whom is to be assigned to one of four subdivisions. Each subdivision will get at least one new employee. In how many ways can these assignments be made ? **(7 Marks)**

8. (a) The number of virus affected files in a system is 1,000 and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(6 Marks)**

(b) Solve the recurrence relation.

$$a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad n \geq 2$$
given $a_0 = 5, a_1 = 12$. **(7 Marks)**

(c) Find the generating function for the recurrence relation

$a_{n+2} - 5a_{n+1} + 6a_n = 2, \ n \geq 0$ and $a_0 = 3, \ a_1 = 7$. Hence solve it. **(7 Marks)**

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CS42

USN

NEW SCHEME

Fourth Semester B.E. Degree Examination, July 2006

CS / IS

Graph Theory and Combinatonics

Time: 3 hrs.] [Max. Marks:100

Note: *1. Answer any FIVE questions choosing at least TWO full questions from each of the parts A and B.*

PART - A

1 a. i) Determine $|V|$, given that $G=(V,E)$ is regular with 15 edges.

ii) Let $G=(V,E)$ be a connected undirected graph. What is the largest possible value for $|V|$ if $|E|=19$ and $\deg(v)\geq 4$ for all $v\in V$? (07 Marks)

b. Define isomorphism of graphs. Show that the following graphs are isomorphic. (07 Marks)

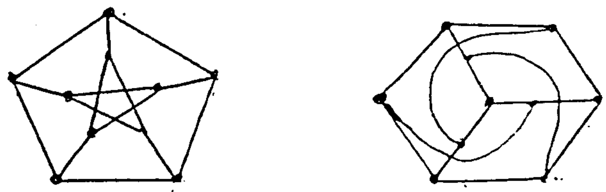

Fig. 1(b)

c. Define :

i) Euler circuit

ii) Hamilton cycle and

iii) Hamilton path.

Give one examples for each. (06 Marks)

2 a. Define a planar graph. Show that the complete graph K_5 (Kuratowski's first graph) is non planar. (07 Marks)

b. Find the geometric dual of the following graph. Write down any four observations of the graph Fig. 2(b) and its dual. (07 Marks)

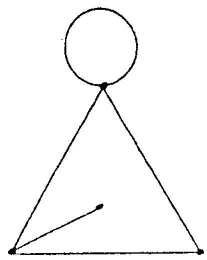

Fig. 2(b)

c. Define chromatic number of a graph. Find the chromatic polynomial $P(G,\lambda)$, where G is a cycle of length four. Hence find the chromatic number. (06 Marks)

3 a. Define a tree. Prove that, if $G = (V, E)$ is an undirected graph, then G is connected if and only if G has a spanning tree. (07 Marks)

b. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur (in a given sample) with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3. (07 Marks)

c. Define :
   - i) Rooted tree
   - ii) Balanced tree and
   - iii) Prefix code.

   Give one example for each. (06 Marks)

4 a. Define :
   - i) cutset
   - ii) Bridge
   - iii) Edge connectivity and
   - iv) Vertex connectivity.

   Give one example for each. (07 Marks)

b. State Kruskal's algorithm. Using Kruskal's algorithm, find a minimal spanning tree for the weighed graph shown in Fig. 4(b).

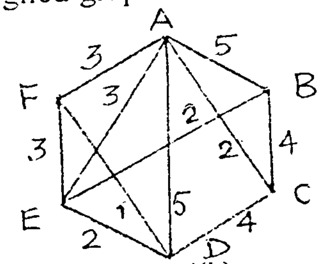

Fig. 4(b)

c. For the network shown in Fig. 4(c), find the capacities of all the cutsets between the vertices a and z and hence determine the maximum flow between a and z. (06 Marks)

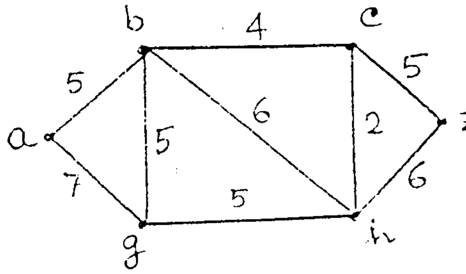

Fig. 4(c)

PART - B

5 a. A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C++, the other four with Java. In how many ways can the professor arrange these books on the shelf
   - i) If there are no restrictions.
   - ii) If the languages should alternate.
   - iii) If all the C++ books must be next to each other.
   - iv) If all the C++ books must be next to each other and all the Java books must be next to each other. (07 Marks)

Contd... 3

b. I) A woman has 11 colleagues in her office, of which 8 are men. She would like to invite some of her colleagues to dinner. Find the number of her choices if she decides to invite :

i) At least 9 of them.

ii) All her women colleagues and sufficient men colleagues to make the numbers of women and men equal.

II) In how many ways can we distribute seven apples and six oranges among four children, so that each child receives at least one apple? (07 Marks)

c. Define Catalan number. In how many ways can one arrange three 1's and three – 1's, so that all six partial sums (starting with the first summand) are non negative? List all the arrangements. (06 Marks)

6 a. State the Pigeonhole principle and the extended Pigeonhole principle. Show that if any 5 numbers from 1 to 8 are chosen, then two of them will have their sum equal to 9. (07 Marks)

b. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? (07 Marks)

c. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. (06 Marks)

7 a. Using the summation operator, find a formula to express $0^2 + 1^2 + 2^2 + \ldots + n^2$ as a function of n. (07 Marks)

b. Use generating function to determine in how many ways can a police captain distribute 24 rifle shells to four police officers, so that each officer gets at least three shells, but not more than eight. (07 Marks)

c. A ship carries 48 flags, 12 each of the colours red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. Use exponential generating function to determine :

i) How many of these signals use an even number of blue flags and an odd number of black flags?

ii) How many of the signals have at least three white flags or no white flags at all? (06 Marks)

8 a. A bank pays 6% annual interest on savings, compounding the interest monthly. If a person deposits \$ 1000 on the first day of May, how much will this deposit be worth a year later? (06 Marks)

b. Solve the recurrence relations :

i) $a_{n+2} + 4a_{n+1} + 4a_n = 7, \quad n \geq 0, \; a_0 = 1, \; a_1 = 2.$

ii) $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, \quad n \geq 0, \; a_0 = 0, \; a_1 = 1.$ (08 Marks)

c. Find the generating function for the recurrence relation.

$a_{n+1} - a_n = 3^n, \; n \geq 0, \; a_0 = 1.$

Hence solve it. (06 Marks)

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NEW SCHEME

CS42

Reg. No. 4VP03CS006

Fourth Semester B.E. Degree Examination, January/February 2006

Computer Science and Information Science Engineering

Graph Theory & Combinatorics

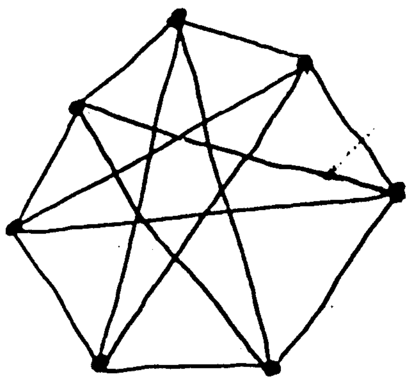
Time: 3 hrs.) (Max.Marks : 100

Note: ***Answer FIVE full questions choosing atleast TWO full questions from each Part.***

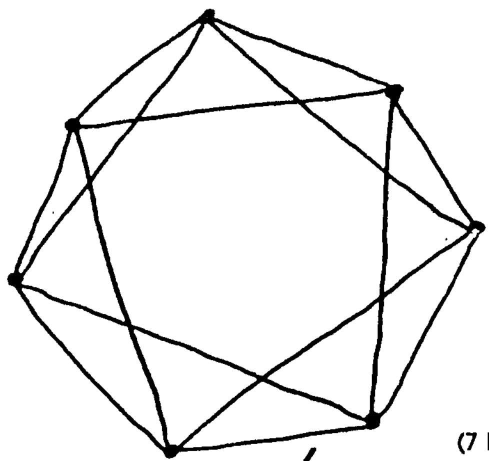
PART - A

1. (a) Determine $|V|$ for the following graphs G.
   i) G has nine edges and all vertices have degree 3.
   ii) G has 10 edges with two vertices of degree 4 and all others of degree 3. **(6 Marks)**

   (b) Define isomorphism of graphs. Show that the following two graphs are isomorphic.



G_1



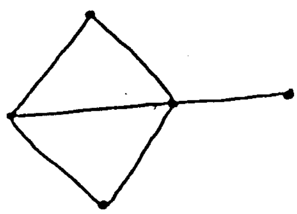
G_1'

(7 Marks)

   (c) Define Hamilton cycle.
   Prove that in the complete graph with n vertices, where n is odd and ≥ 3, there are $\frac{n-1}{2}$ edge - disjoint Hamilton cycles. **(7 Marks)**

2. (a) Define i) a planar graph ii) a bipartite graph, and iii) a complete bipartite graph. Give one example for each **(6 Marks)**

   (b) Find the geometric dual of the graph $G = (V, E)$. Write down any four observations of G and its dual.

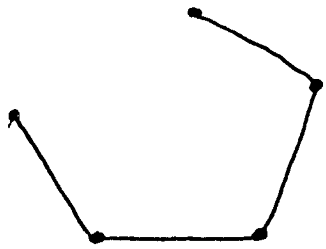


G_1

(7 Marks)

Contd.... 2

(c) Define chromatic number of a graph.
Find the chromatic polynomial $P(G, \lambda)$ for the following graph G. Hence find the chromatic number.



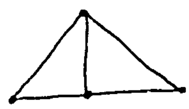
(7 Marks)

3. (a) If a tree $T = (V, E)$ has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, find the number of pendant vertices in T.
(6 Marks)

(b) Construct an optimal prefix code for the symbols

a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7, respectively.
(7 Marks)

(c) Define spanning tree of a graph. Find all the spanning trees of the following graph.

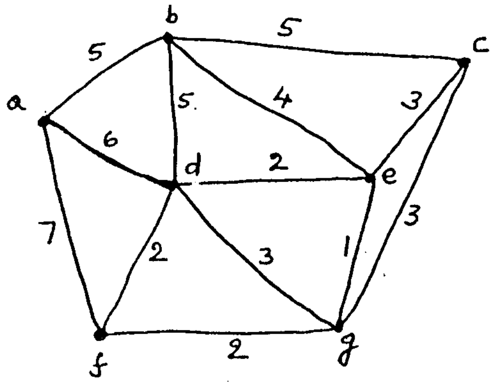


G

(7 Marks)

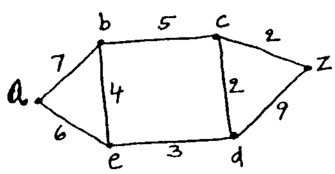
4. (a) Define i) Cutset, ii) Edge - connectivity, and iii) vertex connectivity. Give one example for each.
(6 Marks)

(b) Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below :



(7 Marks)

(c) For the network shown below, find the capacities of all the cutsets between the vertices a and Z, and hence determine the maximum flow between a and Z.



(7 Marks)

Part -- B

8 (2)

PART - B

5. (a) How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000 ? **(6 Marks)**

(b) Consider the following program segment, where i, j and k are integer variables.

```
for i: = 1 to 20 do
  for j : = 1 to i do
    for k : = 1 to i do
        print (i * j + k)
```

How many times is the print statement executed in this program segment ? **(7 Marks)**

(c) Use catalar numbers to find in how many ways can one arrange four 1's and four -1's so that all eight partial sums (starting with the first summand) are non-negative? List all the arrangements. **(7 Marks)**

6. (a) State the Pigeonhole principle and the extended pigeonhole principle. Show that if any six numbers from 1 to 9 are chosen then two of them will add up to 10. **(6 Marks)**

(b) In a certain area of the countryside, there are five villages a, b, c, d, e . An engineer is to device a system of two-way roads so that, after the system is completed, no village will be isolated. In how many ways can he do this ? **(7 Marks)**

(c) Define derangement. In how many ways we can arrange the numbers 1, 2, 3, ... 10 so that 1 is not in 1^{st} place, 2 is not in 2^{nd} place and so on and 10 is not in 10^{th} place. **(7 Marks)**

7. (a) Determine the generating function of the numeric function

$$a_r = \begin{cases} 2^r & if\ r\ is\ even \\ -2^r & if\ r\ is\ odd \end{cases}$$ **(6 Marks)**

(b) Use generating function to determine how many four element subsets of $S = \{1, 2, 3, ...15\}$ contain no consecutive integers. **(7 Marks)**

(c) A company hires 11 new employees, each of whom is to be assigned to one of four subdivisions. Each subdivision will get at least one new employee. In how many ways can these assignments be made ? **(7 Marks)**

8. (a) The number of virus affected files in a system is 1,000 and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(6 Marks)**

(b) Solve the recurrence relation.

$a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad n \geq 2$
given $a_0 = 5, a_1 = 12.$ **(7 Marks)**

(c) Find the generating function for the recurrence relation

$a_{n+2} - 5a_{n+1} + 6a_n = 2, \quad n \geq 0$ and $a_0 = 3, \quad a_1 = 7$. Hence solve it. **(7 Marks)**

** * **

L-52

9 (T)

USN

NEW SCHEME

Fourth Semester B.E. Degree Examination, Dec.06 / Jan.07

CS / IS

Graph Theory and Combinatorics

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer FIVE full questions choosing at least two full questions from each part.***

PART – A

1 a. Define the following terms with respect to graph,

i) Directed graph ii) A Walk iii) Sub-graph

iv) Connected graph v) Multi-graph vi) Simple-graph. **(06 Marks)**

b. What is Konigsberg Bridge Problem? Explain. Draw also the graph for the above bridge. **(04 Marks)**

c. For the figure shown below, compute the degree of each vertex. And also, state which is pendent vertex in a given graph. **(04 Marks)**

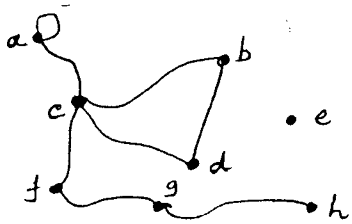


Fig. Q1 (c)

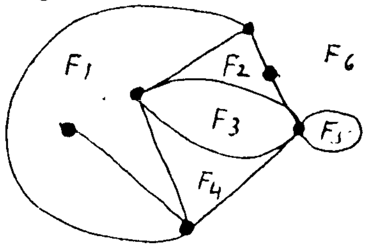
d. Explain the following term with respect to graph :

i) Complete graph. ii) Eularian graph.

iii) Hamiltonian paths and cut. iv) Regular graph. **(06 Marks)**

2 a. What are bi-partite graphs? Explain. Take Petersen graph, then draw a sub-graph of this Petersen graph isomorphic to $K_{3,3}$. **(06 Marks)**

b. Draw the geometric dual of the following given graph. **(04 Marks)**



F_i's i = 1 to 6 are six regions of graph

Fig Q2 (b)

c. What is chromatic number? Explain.
Given the following, A 3-chromatic graph (shown in Fig. Q2 (c)), Compute the chromatic polynomial for this graph. **(04 Marks)**

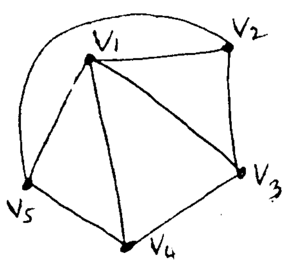


Fig. Q2 (c)

d. Explain the various steps involved in the detection of the planarity of the graph, and hence apply these steps to the graph shown below (Fig. Q2 (d)) to get the final graph. **(06 Marks)**

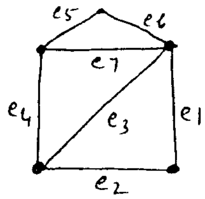


Fig. Q2 (d)

3 a. Define a tree, and also discuss some properties of a tree. **(04 Marks)**

b. Define a rooted tree and a binary tree. Sketch a 13-vertex, 4-level binary tree. **(06 Marks)**

c. When a tree T is said to be a 'Spanning Tree'? Explain. For the following weighted graph, show by darkened lines, a shortest spanning tree, and hence calculate its weight. **(06 Marks)**

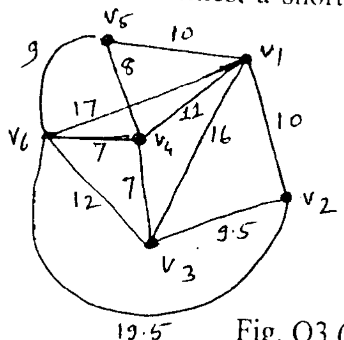


Fig. Q3 (c)

d. What is a "Prefix Code"? For the prefix code P = { 111, 0, 1100, 1101, 10}, draw the labeled full binary tree of height 4. **(04 Marks)**

4 a. Explain the steps involved in Kruskal's Algorithm. For the following graph, get a spanning tree of minimal weight. **(08 Marks)**

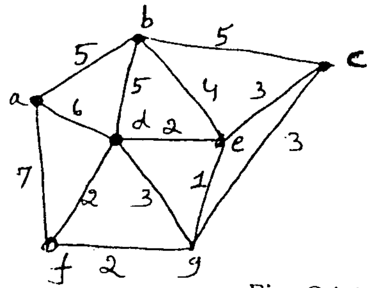


Fig. Q4 (a)

b. Explain the "Labelling Procedure" that is required in the "Max-Flow Min-Cut theorem". And apply this, to the following transportation network to obtain the Max-Flow. **(08 Marks)**

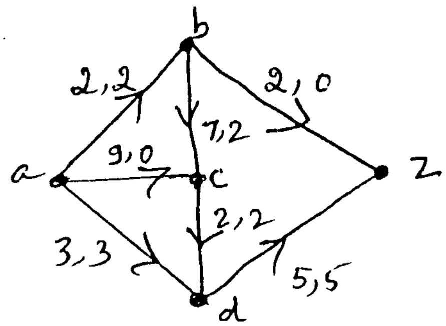


Fig. Q4 (b)

c. What are cutsets? Explain. **(04 Marks)**

Contd. 3

PART - B

5 a. Determine the number of positive integers n where $1 \le n \le 10$ and n is not divisible by 2, 3 or 5. Give the list of those positive integers. **(06 Marks)**

b. State the "Pigeonhole Principle". Triangle ACE is equilateral with AC = 1. If five points are selected from the interior of the triangle, show that there are at least two points whose distance apart is less than ½. **(04 Marks)**

c. Define a "Rook Polynomial". Consider the following Chess-board shown in Figure Q6 (c).

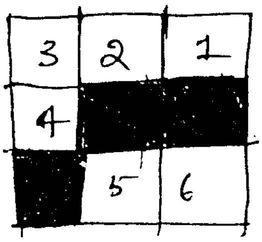


Fig. Q6 (c)

Compute the Rook Polynomial $r(C,x)$ for the above chess board given in Figure Q6 (c). **(04 Marks)**

d. While at the race-track, Ralph bets on each of the ten horses in a race to come in according to how many are favored. In how many ways, can they reach the finish time, so that he loses all of his bets? **(06 Marks)**

6 a. Explain, what is "Binomial Theorem"? Obtain the coefficient of a^5b^2 in the expansion of $(2a-3b)^7$. **(04 Marks)**

b. Solve the following examples :

i) Obtain the coefficient of $x^2y^2z^3$ in the expansion of $(x + y + z)^7$.

ii) A donut shop offers 20 kinds of donuts. Assuming that there are at least dozen of each kind, when we enter the shop, in how many ways can we select a dozen donuts? **(04 Marks)**

c. Write short notes on the following :

i) Ramsey numbers. ii) Partition of integers.

iii) Stirling numbers, and iv) Bell numbers. **(12 Marks)**

7 a. Obtain the generating function for the sequence 0, 2, 6, 12, 20, 30, 42, ... **(04 Marks)**

b. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ where $n \ge 0$ and $F_0 = 0$, and $F_1 = 1$. **(06 Marks)**

c. Solve the recurrence relation $a_n - 3a_{n-1} = 5\,(7^n)$ where $n \ge 1$ and $a_0 = 2$. **(06 Marks)**

d. Find the exponential generating function for the sequence 0!, 1!, 2!, 3! ..... **(04 Marks)**

8 a. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}

i) How many functions are there from A to B?

ii) How many of these are one-to-one?

iii) How many are onto? **(09 Marks)**

b. What are Catalan numbers? Consider the moves R : $(x, y) \longrightarrow (x+1, y)$ and U : $(x, y) \longrightarrow (x, y+1)$

In how many ways can one go,

i) from (0, 0) to (6, 6) and not rise above the line $y = x$?

ii) from (2, 1) to (7, 6) and not rise above the line $y = x - 1$?

iii) from (3, 8) to (10, 15) and not rise above the line $y = x + 5$? **(05 Marks)**

c. Give a combinatorial argument to verify that $\forall n \in Z^+$,

$$n! = \binom{n}{0}d_0 + \binom{n}{1}d_1 + \ldots\ldots + \binom{n}{n}d_n = \sum_{k=0}^{n}\binom{n}{k}d_k$$

(for each $1 \le k \le n$, d_k = the number of derangements of 1, 2, ..., k ; $d_0 = 1$) **(06 Marks)**

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USN | | | | | | | | | | | 06CS42

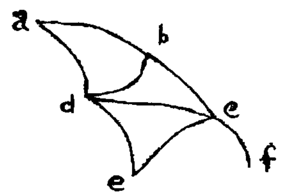
Fourth Semester B.E. Degree Examination, June / July 08
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, selecting at least TWO questions from each PART.***

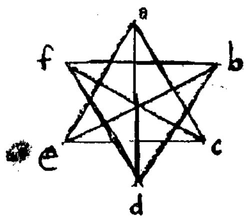
PART - A

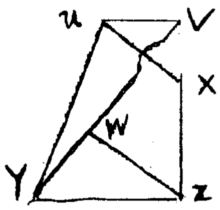
1 a. In the undirected graph



Find i) an a-a circuit of length 6 ii) an a-a cycle of maximum length. **(06 Marks)**

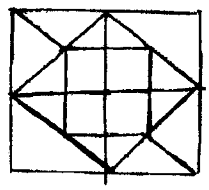
b. Determine whether the following graphs are isomorphic or not. **(08 Marks)**





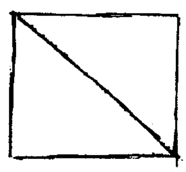
c. Examine if the following graphs are planar or nonplanar : i) K_4 , ii) $K_{3,3}$. **(06 Marks)**

2 a. Find the number of vertices, edges and regions for the following planar graph and verify that Euler's Theorem for connected planar graphs is satisfied : **(04 Marks)**



b. Seven students of a class have lunch together at a circular table. Using Hamilton cycles, determine the minimum number of days required for each of them to sit next to every member of the class. **(10 Marks)**

c. Find the chromatic polynomial for



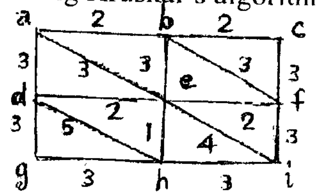
If 4 colours are used, in how many ways can the graph be properly coloured? **(06 Marks)**

3 a. For every tree T = (V,E), if $|V| \geq 2$, show that T has at least two pendant vertices. **(06 Marks)**

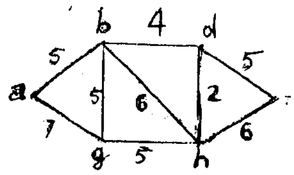
b. A classroom has 25 microcomputers that must be connected to a wall socket that has four outlets. Connections are made using extension cords that have four outlets each. What is the least number of cords needed to get the computers set up for the class? **(06 Marks)**

c. Obtain an optimal prefix code for the symbols that occur with frequencies :
78, 16, 30, 35, 125, 31, 20, 50, 80, 3. **(08 Marks)**

4 a. Obtain a minimal spanning tree using Kruskal's algorithm for (08 Marks)



b. For the given betwork



Consider the two cuts : cut C_1 with vertex partition Q = {a, b, g}, Q' = {d, h, z} and ii) cut C_2 with vertex partition Q = {a, b, d, g, h}, Q' = {z}. Find the capacity of each of these cuts. (04 Marks)

c. Students p, q, r, s, t are members of three committees A, B, and C; r and s belong to committee A; p, r, t belong to committee B and p, q and t belong to committee C. Each committee is to select a student representative. Use the Max-flow-Min-cut theorem to determine if a selection be made such that each committee has a distinct representative? (08 Marks)

PART - B

5 a. Maruti cars come in four models, twelve colours, three engine types and two transmission types. How many distinct Maruti cars can be manufactured? Of these how many have the same colour? (06 Marks)

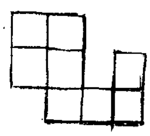
b. Find the coefficient of $x^2 y^2 z^3$ in $(3x - 2y - 4z)^7$. (06 Marks)

c. In how many ways can one distribute eight identical marbles in four distinct containers : i) so that no container is empty ii) so that the fourth container has an odd number of marbles in it.. (08 Marks)

6 a. How many nonnegative integer solutions are there of the equation $x_1 + x_2 + x_3 + x_4 = 18$, where $x_i \geq 7$ for i = 1, 2, 3, 4. (08 Marks)

b. Seven books are distributed among seven students for reading. The books are collected and redistributed. In how many ways will each student get to read two different books? (06 Marks)

c. Find the rook polynomial for



(06 Marks)

7 a. In how many ways can 24 apples be distributed to four children, so that each gets at least three but not more than eight. (06 Marks)

b. Using generating functions, show that the number of partitions of a positive integer into distinct summands is equal to the number of partitions into odd summands. (06 Marks)

c. In how many ways can four of the letters in HAWAII be arranged? (08 Marks)

8 a. Solve the recurrence relation : $a_{n+2} + 3a_{n+1} + 2a_n = 3^n$, $a_0 = 0$, $a_1 = 1$. (06 Marks)

b. Solve using the method of generating function : $a_{n+1} - 3a_n = n$, $a_0 = 1$. (08 Marks)

c. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Show that $A^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$, where F_n is the n^{th} Fibonacci number. (06 Marks)

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06CS42

Fourth Semester B.E. Degree Examination, Dec.08 / Jan.09
Graph Theory and Combinotrics

Time: 3 hrs. Max. Marks:100

Note : ***Answer FIVE full questions, selecting at least two questions from each part.***

Part A

1 a. Give an example of a connected graph G where removing any edge of G results in a disconnected graph. **(03 Marks)**

b. Define homomorphism of a graph. Show that following graphs are isomorphic. **(05 Marks)**

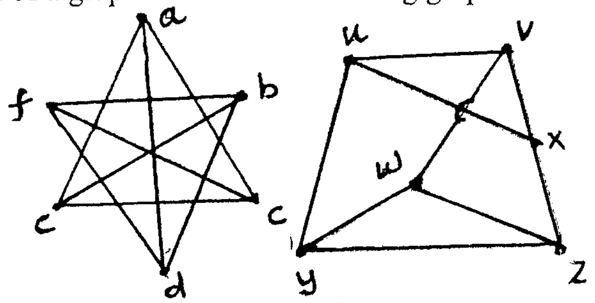

Fig. Q1 (b)

c. Determine $|v|$ for the following graphs or multigraphs G.

i) G has nine edges and all vertices have degree 3.

ii) G has ten edges with two vertices of degree 4 and all other of degree 3. **(04 Marks)**

d. Define, with one example for each: i) Regular graph ii) Complement of a graph iii) Euler trail and Euler circuit. iv) Complete graph. **(08 Marks)**

2 a. Let G = (V, E) be a loop-free connected planar graph with $|v| = v$ and $|E| = e > 2$ and r regions. Then show that $3r \le 2e$ and $e \le 3v - 6$. Using the above relation, show how k_5 and $k_{3,3}$ are nonplanar. **(06 Marks)**

b. Find the dual graph for the following planar graph shown in figure Q2 (b). Write down any four observations of the graph given below and its dual. **(04 Marks)**

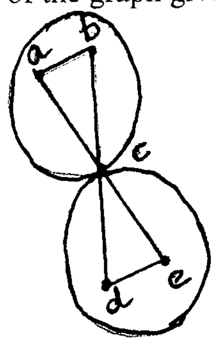

Fig. Q2 (b)

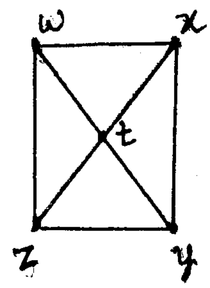

Fig. Q2 (d)

c. Show that Peterson graph has no Hamilton cycle but it has a Hamilton path. **(04 Marks)**

d. Define chromatic number of a graph. Find the chromatic polynomial for the graph shown below and also find the chromatic number for the same. **(06 Marks)**

3 a. Define a Tree. Prove that if $G = (V,E)$ is an undirected graph then G is connected if and only if G has a spanning tree. **(06 Marks)**

b. Define : i) Binary Rooted tree ii) Prefix code iii) Balanced tree. Give one example for each. **(06 Marks)**

c. Construct an optimal prefix code for the symbols a, 0, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(08 Marks)**

4 a. State Kruskal's algorithm and using this algorithm find a minimal spanning tree for the weighted graph shown below. **(06 Marks)**

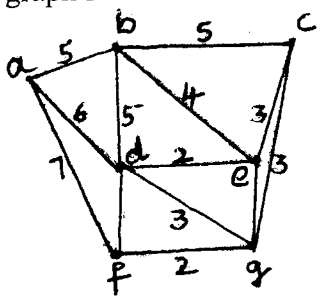

Fig. Q4 (a)

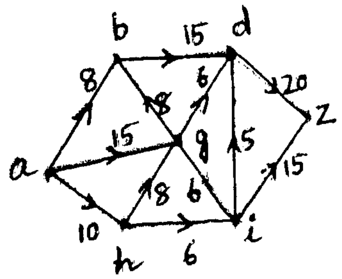

Fig. Q4 (b)

b. Explain Max-flow Min-cut theorem. Apply this to network shown in figure Q4 (b) to obtain the Max-flow. **(08 Marks)**

c. Explain the steps in Dijkstra's shortest path algorithm with example. **(06 Marks)**

Part B

5 a. i) How many arrangements are there for all the letters in sociological.
ii) In how many of the arrangements in part i) are A and G adjacent.
iii) In how many of the arrangement in part i) are all the vowels adjacent. **(06 Marks)**

b. State and explain the meaning of Bionomial theorem. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ and find the sum of all the coefficients in the expansion of $(x+y)^{10}$. **(08 Marks)**

c. Consider the moves: R : (x, y)→(x+1, y), U : (x, y) →(x, y+1).
In how many ways can one go
i) from (0, 0) to (6, 6) and not rise above the line y = x
ii) from (2, 1) to (7, 6) and not rise above the line y = x – 1
iii) from (3, 8) to (10, 15) and not rise above the line y = x + 5 **(06 Marks)**

6 a. Determine the number of +ve integers n where $1 \le n \le 100$ and n is not divisible by 2,3 or 5. **(06 Marks)**

b. Find the number of permutations of a, b……x, y, z in which none of the patterns spin, game, path or net occurs. **(06 Marks)**

c. How many de-arrangements are there for 1, 2, 3, 4, 5? **(04 Marks)**

d. A pair of dice, one is red, the other green is rolled six times. What is the probability that all six values come up on both red die and green die, if the ordered pairs (1, 2) (2, 1) (2, 5) (3, 4) (4, 1) (4, 5) and (6, 6) did not occur. **(04 Marks)**

7 a. If there is an unlimited number (or atleast 24 of each other) of red, green, white or black jelly beans, in how many ways can Douglas select 24 of these, so that he has an even number of white beans and atleast six black ones? **(06 Marks)**

b. Find all partitions of x^7. **(07 Marks)**

c. A ship carries 48 flags, 12 each of the colors red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and odd number of black flags. **(07 Marks)**

8 a. Solve the recurrence relation, **(08 Marks)**
$2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$; $n \ge 0$; $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.

b. Solve the recurrence relation, **(06 Marks)**
$a_{n+2} - 4a_{n+1} + 3a_n = -200$; $n \ge 0$; $a_0 = 3000$, $a_1 = 3300$.

c. Solve the recurrence relation by the method of generating function, **(06 Marks)**
$a_{n+2} - 5a_{n+1} + 6a_n = 2$; $n \ge 0$; $a_0 = 3$, $a_1 = 7$.

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06CS42

Fourth Semester B.E. Degree Examination, June-July 2009
Graph Theory & Combinatorics

Time: 3 hrs.

Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Determine $|\Upsilon|$ for the following graphs:
i) G is regular with 15 edges.
ii) G has 10 edges with two vertices of degree 4 and all others of degree 3. **(05 Marks)**

b. Define isomorphism of graphs. Show that the following graphs are isomorphic.

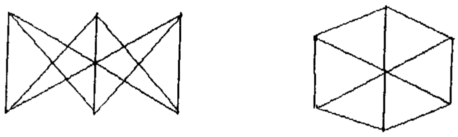

Fig.1(b) **(05 Marks)**

c. Define i) Complete graph ii) Induced subgraph iii) Euler circuit. Give one example for each. **(05 Marks)**

d. If G is an undirected graph with n vertices and e edges, let $\delta = \min_{v\in\Upsilon}\{\deg(v)\}$ and let $\Delta = \max_{v\in\Upsilon}\{\deg(v)\}$, then prove that $\delta \le 2(e/n) \le \Delta$. **(05 Marks)**

2 a. Define Hamilton cycle. How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles. **(05 Marks)**

b. Define : i) Planar graph ii) Bipartite graph iii) Complete bipartite graph. Give one example for each. **(05 Marks)**

c. If G is a connected simple planar graph with n (≥3) vertices, e(>2) edges and r regions, then prove that i) $3r \le 2e$ ii) $e \le 3n - 6$. **(05 Marks)**

d. Define chromatic number. Find the chromatic polynomial for the cycle of length 4 as shown in Fig.2(d) below. Hence find the chromatic number. **(05 Marks)**

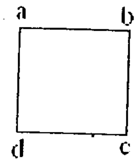

Fig.2(d)

3 a. Define a tree.
i) Prove that a tree with two or more vertices contains at least two pendant vertices.
ii) Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendant vertices in T. **(06 Marks)**

b. Define : (i) Binary rooted tree (ii) Balanced tree.
Draw all the spanning trees of the graph show in Fig.3(b) below. **(07 Marks)**

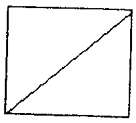

Fig.3(b)

c. Define prefix code. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(07 Marks)**

4 a. Explain Dijkstra's algorithm. (06 Marks)

b. State Kruskal's algorithm. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown in Fig.4(b) below. (07 Marks)

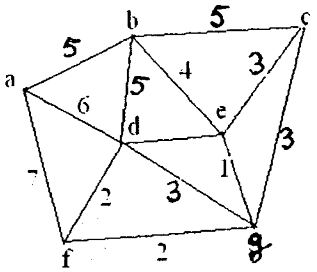

Fig.4(b)

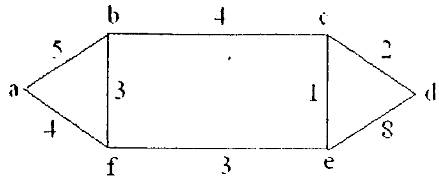

Fig.4(c)

c. Define a cut-set. For the network shown in Fig.4(c), find the capacities of all the cutsets between the vertices a and d, and hence determine the maximum flow between a and b. (07 Marks)

PART – B

5 a. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? (05 Marks)

b. In how many ways can 10 identical dimes be distributed among five children if (i) there are no restrictions (ii) each child gets at least one dime (iii) the oldest child gets at least two dimes. (05 Marks)

c. Determine coefficient of xyz^2 in the expansion of $(2x - y - z)^4$. (05 Marks)

d. Define Catalan number. Using the moves $R : (x, y) \to (x+1, y)$ and $v : (x, y) \to (x, y+1)$, find in how many ways can one go

i) From (2,1) to (7,6) and not rise above the line $y = x - 1$.

ii) From (3,3) to (10,15) and not rise above the line $y = x + 5$. (05 Marks)

6 a. How many integers between 1 and 300 (inclusive) are

i) divisible by at least one of 5, 6, 8?

ii) divisible by none of 5, 6, 8? (06 Marks)

b. Define derangement. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. (07 Marks)

c. Find the rook polynomial for the 3×3 board using the expansion formula. (07 Marks)

7 a. i) Find a generating function for the sequence $1^2, 2^2, 3^2, \ldots\ldots$

ii) Find the coefficient of x^n in the expansion of $(x^2 + x^3 + x^4 + \ldots\ldots)^4$ (06 Marks)

b. Use generating function to determine in how many ways can two dozen identical robots be assigned to four assembly lines with i) at least 3 robots assigned to each line ii) at least 3 but not more than 8 robots assigned to each line. (07 Marks)

c. Using exponential generating function, find the number of ways in which 4 of the letters in ENGINE be arranged. (07 Marks)

8 a. Find and solve a recurrence relation for the number of binary sequences of length $n \geq 1$ that have no consecutive 0's. (06 Marks)

b. Solve the recurrence relation

$$a_{n+2} + 3a_{n+1} + 2a_n = 3^n \quad \text{for} \quad n \geq 0; \quad \text{given } a_0 = 0, \ a_1 = 1.$$ (07 Marks)

c. Find a generating function for the recurrence relation

$a_{n+2} - 2a_{n+1} + a_n = 2^n$ for $n \geq 0$; given $a_0 = 1$, $a_1 = 2$.

Hence solve it. (07 Marks)

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06CS42

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Determine $|V|$ for the following graphs.
   i) G has nine edges and all vertices have degree 3.
   ii) G is regular with 15 edges. **(06 Marks)**

b. Define isomorphism of graphs. Show that no two of the following three graphs as shown in Fig.1(b) are isomorphic. **(07 Marks)**

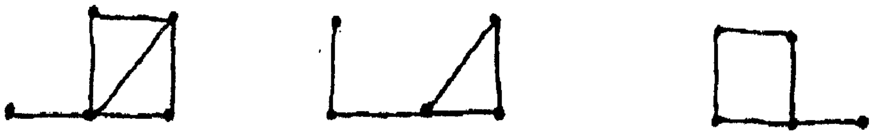

Fig.1(b).

c. Define Euler circuit. Discuss Konigsberg bridge problem. **(07 Marks)**

2 a. Define: i) Bipartite graph ; ii) Hamilton cycle and iii) Planar graph. Give one example for each. **(06 Marks)**

b. If G = (V, E) is a loop-free connected planar graph with $|V| = n$, $|E| = e > 2$, and r regions, then prove that : i) $e \geq \frac{3r}{2}$; ii) $e \leq 3n - 6$. Further, if G is triangle free, then iii) $e \leq 2n - 4$. **(07 Marks)**

c. Define chromatic number. Find the chromatic polynomial for the cycle of length 4. Hence find its chromatic number. **(07 Marks)**

3 a. Define a tree. Prove that in every tree $T = (V, E)$, $|V| = |E| + 1$. **(06 Marks)**

b. Define: i) Rooted tree ; ii) Complete binary tree and iii) Spanning tree. Give an example for each. **(07 Marks)**

c. Obtain an optimal prefix code for the message ROAD IS GOOD using labelled binary tree. Indicate the code. **(07 Marks)**

4 a. State Max-flow and Min-cut theorem. For the network as shown in Fi.4(a), determine the maximum flow between the vertices A and D by identifying the cut-set of minimum capacity. **(06 Marks)**

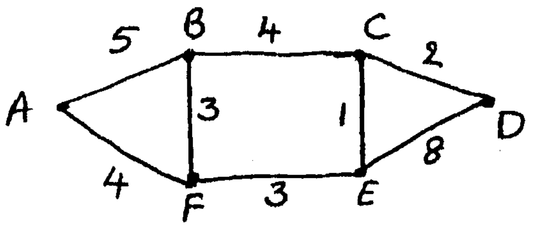

Fig.4(a).

b. State Kruskals algorithm. Apply Kruskal's algorithm to find a minimal spanning tree for the weighted graph as shown in Fig.4(b). **(08 Marks)**

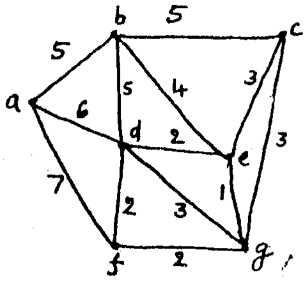

Fig.4(b).

c. Explain the steps in Dijikstra's shortest path algorithm. (06 Marks)

PART – B

5 a. i) How many distinct four digit integers can one make from the digits 1, 3, 3, 7, 7 and 8?
ii) Find the number of arrangements of the letters in TALLAHASSEE which have no adjacent A's (06 Marks)

b. In how many ways can 10 identical dime be distributed among five children if,
i) There are no restrictions
ii) Each child gets at least one dime
iii) The oldest child gets at least two dimes. (07 Marks)

c. Define Catalan number. In how many ways can one arrange three 1's and three – 1's so that all six partial sums (starting with the first summand) are nonnegative? List all the arrangements. (07 Marks)

6 a. Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. (06 Marks)

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. (07 Marks)

c. A girl student has sarees of 5 different colours: blue, green, red, white and yellow. On Mondays, she does not wear green; on Tuesdays, blue or red; on Wednesday, blue or green; on Thursdays red or yellow; on Fridays, red. In how many ways can she dress without repeating a colour during a week (from Monday to Friday)? (07 Marks)

7 a. Find a generating function for each of the following sequences:
i) $1^2, 2^2, 3^2, 4^2, \ldots\ldots\ldots$
ii) 8, 26, 54, 92,.......... (06 Marks)

b. Using the generating function, find the number of ways of forming a committee of 9 students drawn from 3 different classes so that students form the same class do not have an absolute majority in the committee. (07 Marks)

c. If a leading digit 0 is permitted, using exponential generating function, find the number of r – digit binary sequences that can be formed using an even number of 0's and an odd number of 1's. (07 Marks)

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. (06 Marks)

b. Solve the following recurrence relations:
i) $a_n - 3\,a_{n-1} = 5\,(3^n),\ n \ge 1,\ a_0 = 2.$
ii) $a_{n+2} + 4\,a_{n+1} + 4\,a_n = 7,\ n \ge 0,\ a_0 = 1,\ a_1 = 2.$ (08 Marks)

c. Find the generating function for the recurrence relation:
$a_{n+2} - 2\,a_{n+1} + a_n = 2^n,\ n \ge 0$ with $a_0 = 1,\ a_1 = 2$. Hence solve it. (06 Marks)

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06CS42

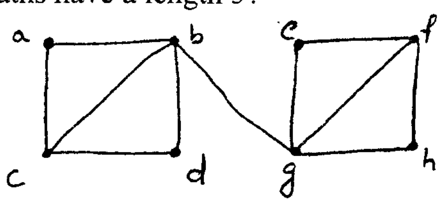
Fourth Semester B.E. Degree Examination, May/June 2010
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Let G = (V, E) be the undirected graph in the Fig.1(a). How many paths are there in G from a to h? How many of these paths have a length 5? **(07 Marks)**

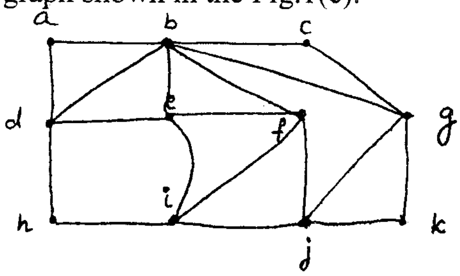
Fig.1(a).



b. Let G = (V, E) be an undirected graph, where $|V| \geq 2$. If every induced subgraph of G is connected, can we identify the graph G? **(06 Marks)**

c. Find an Euler circuit for the graph shown in the Fig.1(c). **(07 Marks)**

Fig.1(c).



2 a. Show that when any edge is removed from K_5, the resulting subgraph is planar. Is this true for the graph $K_{3,3}$? **(07 Marks)**

b. Nineteen students in a nursery school, play a game each day, where, they hold hands to form a circle. For how many days can they do this, with no student holding hands with the same playmate twice? **(07 Marks)**

c. Define chromatic number. What is chromatic polynomial? State the decomposition theorem for chromatic polynomials. **(06 Marks)**

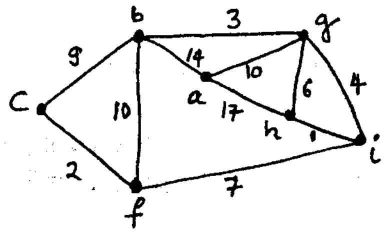
3 a. A classroom contains 25 microcomputers, that must be connected to a wall socket that has four outlets. Connections are made by using extension cords, that have four outlets each. What is the least number cords needed to get these computers set up for class use? **(07 Marks)**

b. Explain the steps in the merge sort algorithm. **(06 Marks)**

c. Using the weights 2, 3, 5, 10, 10, show that the height of the Huffman tree for a given set of weights is not unique. **(07 Marks)**

4 a. Apply Dijkstra algorithm to the weighted graph G = (V, E) shown in Fig.4(a) and determine the shortest distance from vertex a to each of the other vertices in the graph. **(07 Marks)**

Fig.4(a)



b. Use Prim's algorithm to generate an optimal tree for the graph, shown in Fig.4(b). (06 Marks)

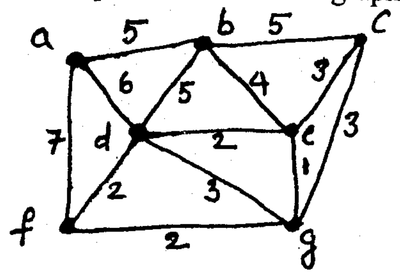


Fig.4(b).

c. Let f be a flow in a network N = (V, E). If C = (P, $\overline{P}$) is any cut in N, then prove that val (f) cannot exceed C (P, $\overline{P}$). (07 Marks)

5 a. In a certain implementation of the programming language Pascal, an identifier consists of a single letter or a letter followed by upto seven symbols, which may be letters or digits. (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of Pascal? (07 Marks)

b. How many bytes contain i) Exactly two 1's ; ii) Exactly four 1's ; iii) Exactly six 1's and iv) At least six 1's? (07 Marks)

c. In how many ways can 10 (identical) dimes be distributed among five children i) If there are no restrictions ; ii) Each child gets at least one dime ; iii) The oldest child gets at least 2 dimes. (06 Marks)

6 a. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2, 3, 5. (07 Marks)

b. In how many ways can one arrange the letters in CORRESPONDENTS so that i) There is no pair of consecutive identical letters ; ii) There are exactly two pairs of consecutive identical letters. (07 Marks)

c. For the positive integers 1, 2, 3, 4, there are n derangements. Define derangements. What is the value of n? (06 Marks)

7 a. Give the generating function for :

i) 1, 1, 1, 1,........1, 1, 1.... all terms are 1

ii) 1, 1, 1, 1,........1, 0, 0, 0..... first terms are 1, others are 0

iii) 0, 1, 2, 3........ (06 Marks)

b. Find the generating function for $P_d(n)$, the number of partitions of a positive integer n into distinct summands. What is $P_d(6) = ?$ (07 Marks)

c. In each of the following, the function f(x) is the exponential generating function for the sequence a_0, a_1, a_2......., whereas the function g(x) is the exponential generating function for the sequence b_0, b_1, b_2....... Express g(x) in terms of f(x) if

i) $b_3 = 3$ and $b_n = a_n$, $n \in N, n \neq 3$.

ii) $b_1 = 2, b_2 = 4$, and $b_n = 2a_n$, $n \in N, n \neq 1, 2$. (07 Marks)

8 a. Solve the following recurrence relation :

$a_n = 5a_{n-1} + 6a_{n-2}$ $n \ge 2$ $a_0 = 1, a_1 = 3$. (10 Marks)

b. Solve the following recurrence relation:

$a_{n+1} - 2a_n = 2^n$, $n \ge 0$ $a_0 = 1$. (10 Marks)

c. Solve the following recurrence relation using the method of generating functions :

$a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \ge 0, a_0 = 3, a_1 = 7$.

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